KINEMATIC HARDENING MODEL COMPARISON OF SQUARE HOLLOW SECTION UNDER CYCLIC BENDING

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ABSTRACT

This study compares the different linearity of the kinematic hardening model of the Square Hollow Section (SHS) under cyclic bending loading. Four specimens of a simple support beam cyclically tested in previous research are listed as hot-rolled, hot-finished, and two cold-formed. Using the bilinear, multilinear, and Chaboche models, each specimen is modeled in kinematic hardening. The variables or node sets for each linearity model are estimated using tensile test data, and Chaboche variables are obtained using the least-square fitting method. Each linearity model for each specimen is built in FEA using a shell model. The numerical model applied the same cyclic loading history as the previous test. The numerical analysis comparison concluded that Chaboche and the multilinear kinematic model generate the expected result fitted to test hysteresis of cold-formed one and cold-formed 2 SHS, but the bilinear models are not fitted. Moreover, all kinematic models are not fit for the hot-rolled and hot-finished SHS compared to the test hysteresis. So, for hot-rolled and hot-finished SHS, the combined hardening is suggested; there is a possibility it is because of the lower yield ratio that both sections have. Overall, during a cyclic bending analysis of cold-formed SHS, multilinear or Chaboche models are preferable if the data is limited.

Keywords: Bilinear; Chaboche Model; Kinematic Hardening; Multilinear; Square Hollow Section

INTRODUCTION

Square Hollow Section (SHS) is one of the preferable steel sections nowadays, based on its functional features of higher restrained capability as a structural member and better architectural consideration (Chavan et al., 2014). The increase in its usage in steel construction led to needed studies about SHS behavior in any loading situation. Understanding the behavior is important to ensure the structural safety of SHS involved in the building construction. The study can include experimental or numerical analysis-based research. For this SHS, many studies are available: during axial loading (Fang & Chan, 2019), torsion loading (Fadden, 2013), monotonic bending (Wang et al., 2016a; Wang et al., 2016b; Gkantou et al., 2018), combined of axial and monotonic bending (Ma et al., 2019; Arrayago & Real, 2015; Meng & Gardner, 2020; Yun et al., 2020; Zhao et al., 2016), but only a few about cyclic bending and its parametric studies (Fadden & McCormick, 2014).

In this article, previous hysteresis data about cyclic bending SHS are available (Terada et al., 2012), including a series of the tensile stress-strain data. Four specimens of different manufacturing forming processes of the same SJ355J2H steel grade are discussed in this article: a hot-rolled, a hot-finished, and two cold-formed SHS. Hot-rolled and hot-finished sections are based on EN 10210 (EN
10210 Hot Finished Structural Hollow Sections of Non-Alloy and Fine Grain Steels, 2006), and two cold-formed SHS sections are based on EN 10219 (EN 10219 Cold-Formed Welded Structural Hollow Sections of Non-Alloy and Fine Grain Steels, 2006). Although the specimens have the same grade material, each has different material properties and produces different load-displacement hysteresis.

The experimental-based studies may obtain more reliable results of required ductility, fracture toughness value, or absorption energy of a subjected element. However, it is challenging in terms of time and cost. Sometimes, numerical analysis can be a sustainable and acceptable alternative that helps the researcher gain more results based on different loading, material properties, or sections dimension in any discipline (Bhashyam, 2002a).

Geometry, loading, and material properties must be defined during numerical analysis. Sequence research also concludes that hardening models are mainly responsible for structural behavior differences between materials (Nip et al., 2010). The difference in hardening model linearity available in the FEA also needs attention because it may lead to a different result, whether bilinear, multilinear or nonlinear.

Isotropic hardening means the yield surface uniformly expands in all directions, correlated with the accumulated dislocation structure during plastic deformation (Lin et al., 2011). As for kinematic hardening, the yield surface translates in the direction of yields but remains constant in size. This hardening model assumes that neither expansion/contraction nor distortion of the yield surface happened in the stress space (Patillo, 2018). Kinematic hardening is essential for predicting cyclic plasticity of unloading and strain reversals modeling (Lin et al., 2011; Chaboche & Rousselier, 1983). In this research about cyclic bending analysis, these kinematic hardening models with different linearity models are used.

This study about cyclic bending of SHS and the hardening model comparison is important for numerical analysis study, especially when limited data for analysis of the member behavior is limited. Another previous research by the author discussed the usage of the Chaboche nonlinear kinematic hardening parameter obtained from the proposed method (Tsuchiya & Ochi, 2016; Nagai & Ochi, 2019; Matshusita et al., 2019). Another article comparing the hardening model applied in the FEA did not discuss cyclic bending analysis (Budaházy & Dunai, 2013). This study provides an alternative insight into the numerical analysis of the linearity of kinematic models, focused on different material forms of SHS cyclic bending numerical analysis.

**METHODS**

The methods used in this research are presented in the flow chart in Figure 1. The initial step in this research starts with prepare collecting experimental data, including geometries, material properties (tensile test), and cyclic bending load-hysteresis data (Terada et al., 2012). The SHS has different dimensions based on the section standard. The hot-rolled and hot-finished SHS are 180×180×12.5 EN 10210 standard section (EN 10210 Hot Finished Structural Hollow Sections of Non-Alloy and Fine Grain Steels, 2006), and the cold-formed 1 and 2 SHS is 200×200×12.5 EN 10219 standard section (EN 10219 Cold-Formed Welded Structural Hollow Sections of Non-Alloy and Fine Grain Steels, 2006).
The material properties of the four sections are listed in.

. The part location is divided into flat and corner, except for hot-rolled SHS with the same material properties on its flat and corner parts. Then, the experimental setup is presented in Figure 2. In Figure 2, a three-meter length beam at 45 degrees has a cyclic loading in the middle of the section. Each of the specimens has the same setup. The incremental cyclic loading history is presented in Figure 3, the displacement load is calculated for each primary displacement ($\delta_p$) in the section.

The next step is estimating the variables of each kinematic linearity model, namely: the bilinear, multilinear, and nonlinear Chaboche model. The defined variables are then used to define material properties in the FEA. Each specimen has different material properties for the different linearity model.

The cyclic bending analysis done in the FEA is based on the test setup in Figure 2. At the same time, the cyclic loading history of Figure 3 used in the FEA was customized as the same as the finished cyclic test data. Four specimens of SHS are modeled in the FEA as cantilever beams to shorten the analysis time. This assumption can be used because the test setup of the previous research is symmetric in the middle of the beam. So, if the test setup has three meters of beam length, the half specimen considered a cantilever model is 1.5 meters long. The model uses a three-dimensional shell model (Shell 181), with the fixed restraint at the end and the cyclic loading at the other end of the beam. For the meshing setting, the mesh near the fixed restraint is tighter to accommodate the possibility of local buckling happening at that part. The illustration of these numerical models in the FEA is presented in Figure 4, and the its three-dimensional model illustrated in Figure 5.

The next part is the hysteresis comparison between the test and the numerical analysis results. The result of FEA for bilinear, multilinear, and Chaboche models was then compared to the experimental data's hysteresis. Then comparison result of this study can then be discussed so that the conclusion can be obtained.

### RESULTS AND DISCUSSION

#### Kinematic hardening model variables

In the kinematic hardening model in many FEA, there are listed linearity models as bilinear, multilinear and nonlinear. The bilinear kinematic hardening model is the simplest hardening that can be used if the data is very limited to yield and ultimate stress.

A constitutive equation to define material variables (Jia & Kuwamura, 2014). Two variables included in the bilinear model are $\sigma_y$ (yield stress) and $E_T$ (Tangent modulus). For four specimens discussed in this article, four sets of bilinear model variables are listed in Table 2. Table 2 informs that hot-rolled SHS has the lowest variables of Tangent modulus. In the hot-finished SHS, the corner parts have a higher value of the Tangent modulus than the flat parts. Meanwhile, both cold-formed SHSs have the highest value of other specimens due to their ultimate strain point that lay on a very small value.

$$E_T = \left(\sigma'_u - \sigma'_y\right) / \varepsilon'_u$$  \hspace{1cm} (1)

Where:

- $E_T$ = Tangent modulus (MPa)
- $\sigma'_u$ = ultimate true stress (MPa)
- $\sigma'_y$ = yield stress (MPa)
- $\varepsilon'_u$ = related plastic strain at ultimate

The next linearity model to be compared is the multilinear model. The multilinear model is applicable in many FEA,
including ANSYS Mechanical APDL (Bhashyam, 2002b). The variables are defined by a series of true plastic strain-stress points in the ascending values. This multilinear model is preferable, as written in some research (Bouchenot et al., 2016; Lakshminarayanan & Technologies, 2014; Resapu & Perumahanthi, 2020). When the data is very limited to yield and ultimate stress, some papers proposed how to estimate the stress-strain graph for hot-rolled material (Gardner et al., 2017; Yun & Gardner, 2017), and cold-formed material (Gardner & Yun, 2018).

In the ANSYS Mechanical APDL Release 13's element reference, the multilinear strain-stress set can be defined for a kinematic model of up to five sets (Ansys inc., 2010). The usage of the newer Academic version of ANSYS Mechanical APDL 2021 affects the hardening model available in ANSYS, Moreover, the nonlinear kinematic hardening model available in Ansys Mechanical APDL is the Chaboche's model (Chaboche, 1986, 2008). Chaboche's model has a better nonlinear formulation than another simplistic approach, such as Mroz (Wu et al., 2016). The hardening model has been available in the rate-independent von-Mises kinematic model of ANSYS Mechanical APDL since the 5.6 version. In the ANSYS's note (Imaoka, 2008; ANSYS, 2005), three fitting methods are available to define Chaboche's parameter values, listed as using several stabilized cycles, one stabilized cycle, and using a single tensile test data.

The method used in this article is a single tensile test data. This approach may suit situations dealing with a few cycles like this cyclic bending analysis. Jia's research also uses tensile tests data only to estimate the hardening variables (Jia & Kuwamura, 2014). To begin with, the true plastic stress needed to be converted to back stress ($\alpha_i$) data for any equivalent plastic strain ($\varepsilon_{pl}$), to simplify the curve fitting method, the plastic true stress data used is from the multilinear model node sets of Figure 6.

The backstress of each point ($\alpha_i$) was obtained by subtracting the true plastic stress ($\sigma$) of the scalar values of the initial yield surface. The initial yield surface ($\sigma_{y0}$) for this Chaboche's hardening rules is assumed as equivalent to the yield stress ($\sigma_y$), as written in Eq. (2)

$$\sigma_y = \sigma_{y0} \quad (2)$$

The next step is that the value of each backstress data set should agree to Eq. (3) for one backstress Chaboche and Eq. (4) for two backstress Chaboche. The least-square fitting method in MATLAB (Crișan, 2016) generates Chaboche variables of the specimens presented in Figure 7 for the hot-rolled and hot-finished SHS and Figure 8 for the cold-formed SHS. The material variables are listed in Table 3, the material variables can be used in numerical analysis.

$$\alpha_i = \frac{C}{\gamma} (1 - e^{-\gamma \varepsilon_{pl}}) \quad (3)$$

$$\alpha_i = \frac{C_1}{\gamma_1} (1 - e^{-\gamma_1 \varepsilon_{pl}}) + C_2 \varepsilon_{pl} \quad (4)$$

Where:
$\alpha_i$ = the backstress at of kinematic hardening

$C$ = the modulus of kinematic hardening

$\gamma$ = rate of the cyclic stabilization

$C_1$ = the first modulus of kinematic hardening

$\gamma_1$ = rate of the cyclic stabilization

$C_2$ = the second modulus of kinematic hardening

Figure 7 shows the least-square curve fitting result of Eq. (3) to backstress data of hot-rolled and hot-finished based SHS sections. As seen in Figure 7 (a) and the value listed in Table 3, the Chaboche variables of hot-rolled material tend to have a linear value, as indicated by the very low rate of cyclic stabilization ($\gamma$) value. The almost linear value fitted might be because of the yield plateau of those materials.

Figure 7 (b) and (c) show the curve fitting process for hot-finished SHS. It similarly has a small value cyclic stabilization rate ($\gamma$). Whereas, fitting to the Eq. (4) for hot-rolled and hot-finished based SHS resulting negative value, so that in Table 3, only one backstress is listed. Moreover, Figure 8 presents cold-formed 1 and 2 SHS curve fitting of one and two backstress Chaboche models. The Chaboche model variables for Cold-formed SHS 1 and 2 are listed in Table 3. The two backstress Chaboche model fits better to backstress node sets.

**Numerical Hysteresis Comparison to Previous Research**

Numerical results test generated load-displacement hysteresis of each linearity model and each SHS specimen. The hysteresis of FEA results of each linearity model was then compared to experimental results, as presented in Figure 9.

Figure 9 (a) presents a hysteresis comparison of hot-rolled SHS accordingly. All the linearity models of kinematic hardening applied for the hot-rolled SHS underestimate the hysteresis values, except for the first cycle. Also, in Figure 9 (a), all the linearity models generate almost the same values for each cycle.

The same unfit pattern in hot-finished SHS happened in Figure 9 (b). Each linearity model produces a different hysteresis. One backstress Chaboche model generates a higher load value than the bilinear and multilinear models. The kinematic model cannot capture the loading gap in each cycle, which is usually induced by isotropic hardening. In conclusion, a kinematic model only may not be enough to build a model of the hot-rolled and hot-finished SHS sections. A combined hardening parameter may be required to be involved during numerical analysis.

Figure 9 (c) and (d) present the cold-formed 1 and 2 comparisons to experimental hysteresis. The cold-formed 1 and 2 analysis result shows an expected result than hot-rolled and hot-finished SHS, especially the multilinear and Chaboche model.

Chaboche model shows different fitting lines in but generates relatively similar hysteresis values as presented in Figure 9 (c) and (d). The multilinear kinematic model also gives the same hysteresis as the Chaboche model. However, the bilinear model of cold-formed 1 and 2 SHS has less value than Chaboche and the multilinear model. These patterns may encourage the usage of multilinear mode instead of the bilinear model. Overall, the Chaboche and the multilinear models are preferable for modeling cold-formed SHS sections due to the relatively similar value presented.

As state in the introduction, there are no research that has certain similar topic as this study. The only relatively correlated is the comparison research about different hardening usage to axial cyclic
CONCLUSION

In this study, four specimens of SHS are listed as hot-rolled, hot-finished two cold-formed. Numerical analysis for each specimen SHS with different kinematic models compared to experimental hysteresis. Based on linearity, three different models can be applied in kinematic hardening: the bilinear, multilinear, and Chaboche models. The bilinear model is the simplest in the FEA; however, this model generates underestimated hysteresis compared to previous test data in this study. Multilinear model node sets are taken from the true stress strain of tensile test data. Chaboche model variables were estimated using the least-square fitting method for each specimen of SHS. All kinematic linearity models did not produce a well-fitted result of hot-rolled and hot-finished SHS. It may need combined hardening to be involved instead. In comparison, Chaboche and the multilinear model generate the expected result of hysteresis for cold-formed 1 and 2 SHS. Overall, during a cyclic bending analysis of cold-formed based material, Chaboche or multilinear model, with the variables estimated using tensile test data is preferable if the data is limited.

ACKNOWLEDGEMENT

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CONFLICT OF INTEREST

The authors declare no conflict of interest

REFERENCES


EN 10210 Hot Finished Structural Hollow Sections of Non-Alloy and Fine Grain Steels, (2006). EN 10219 Cold-formed welded
structural hollow sections of non-alloy and fine grain steels, (2006).


Appendix

Table 1. Material Properties (Terada et al., 2012)

<table>
<thead>
<tr>
<th>Manufacturing Process</th>
<th>Part Location</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$Y_R$</th>
<th>$\varepsilon_u$</th>
<th>$\sigma_u^t$ (MPa)</th>
<th>$\varepsilon_u^t$</th>
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</thead>
<tbody>
<tr>
<td>Hot-rolled</td>
<td>Flat and Corner</td>
<td>420</td>
<td>538</td>
<td>0.79</td>
<td>0.110</td>
<td>603</td>
<td>0.118</td>
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<tr>
<td>Hot-finished</td>
<td>Flat faces</td>
<td>434</td>
<td>531</td>
<td>0.86</td>
<td>0.099</td>
<td>589</td>
<td>0.106</td>
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<tr>
<td></td>
<td>Corners</td>
<td>567</td>
<td>624</td>
<td>0.91</td>
<td>0.063</td>
<td>665</td>
<td>0.066</td>
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<tr>
<td>Cold-Formed 1</td>
<td>Flat faces</td>
<td>511</td>
<td>535</td>
<td>0.96</td>
<td>0.050</td>
<td>571</td>
<td>0.077</td>
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<tr>
<td></td>
<td>Corners</td>
<td>564</td>
<td>597</td>
<td>0.95</td>
<td>0.008</td>
<td>603</td>
<td>0.017</td>
</tr>
<tr>
<td>Cold-Formed 2</td>
<td>Flat faces</td>
<td>542</td>
<td>568</td>
<td>0.95</td>
<td>0.034</td>
<td>601</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>Corners</td>
<td>573</td>
<td>619</td>
<td>0.93</td>
<td>0.014</td>
<td>631</td>
<td>0.028</td>
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</table>

Table 2. Bilinear kinematic model variables

<table>
<thead>
<tr>
<th>Manufacturing Process</th>
<th>Part Location</th>
<th>$\sigma_y$ (MPa)</th>
<th>$E_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot-rolled</td>
<td>Flat and Corner</td>
<td>420</td>
<td>1578</td>
</tr>
<tr>
<td>Hot-finished</td>
<td>Flat faces</td>
<td>456</td>
<td>1464</td>
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<tr>
<td></td>
<td>Corners</td>
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<td>1550</td>
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<td>805</td>
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<td>Corners</td>
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<td>2737</td>
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<tr>
<td>Cold-Formed 2</td>
<td>Flat faces</td>
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<td>917</td>
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<tr>
<td></td>
<td>Corners</td>
<td>573</td>
<td>2301</td>
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</table>

Table 3. Chaboche kinematic model variables

<table>
<thead>
<tr>
<th>Manufacturing Process</th>
<th>Part Location</th>
<th>One Backstress</th>
<th>Two Backstress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$C$ $\gamma$ $C1$ $C1$ $C2$</td>
<td></td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>Flat and Corner</td>
<td>1879 1.55 - - -</td>
<td></td>
</tr>
<tr>
<td>Hot-finished</td>
<td>Flat faces</td>
<td>2150 6.47 - - -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corners</td>
<td>2512 13.59 - - -</td>
<td></td>
</tr>
<tr>
<td>Cold-Formed 1</td>
<td>Flat faces</td>
<td>2445 38.31 3097 109.56 494</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corners</td>
<td>13597 335.35 13777 353.37 116</td>
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</tr>
<tr>
<td>Cold-Formed 2</td>
<td>Flat faces</td>
<td>3796 65.82 6457 206.7 49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corners</td>
<td>17486 294.55 18385 332.64 216</td>
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Figure 1. Flowchart of the research

Figure 2. Cyclic bending test setup (Terada et al., 2012)

Figure 3. Cyclic loading history (Terada et al., 2012)
Figure 4. Numerical model in FEA

Figure 5. Three-dimensional numerical model in FEA
Figure 6. Bilinear and multilinear kinematic variables (node sets) compared to plastic true strain-stress data
(a) Hot-Rolled SHS; (b) Hot-Finished SHS; (c) Cold-Formed 1 SHS; (d) Cold-Formed 2 SHS
Figure 7. Chaboche model (one backstress) curve fitting to the backstress
(a) Hot-Rolled SHS; (b) Flat part of Hot-Finished SHS; (c) Corner part of Hot-Finished SHS
Figure 8. Chaboche model (one and two backstress) curve fitting to the backstress 
(a) Flat part of Cold-Formed 1 SHS; (b) Corner part of Cold-Formed 1 SHS; 
(c) Flat part of Cold-Formed 2 SHS; (d) Corner part of Cold-Formed 2 SHS
Figure 9. Hysteresis comparison of bilinear, multilinear, and Chaboche model to experimental (Terada et al., 2012)

(a) Hot-Rolled SHS; (b) Hot-Finished SHS;
(c) Cold-Formed 1 SHS; (d) Cold-Formed 2 SHS